COURSE ANNOUNCEMENT
MODEL THEORY AND COMBINATORICS
MATH 223M, UCLA, FALL 2016
MWF 10:00AM–10:50AM, MS 5148

Instructor: Artem Chernikov
E-mail: chernikov@math.ucla.edu
Office: MS 5372 (Mathematical Sciences Building)
Office phone: (310) 825-1486
Office hours: M 3:00 pm - 5:00 pm

Description. This is a topics course on applications of model theory in graph (and hypergraph) combinatorics, concentrated on the *regularity phenomena* (Szemerédi's regularity lemma and its relatives). We will give simple proofs of the regularity and removal lemmas for graphs and hypergraphs using probability measures on ultraproducts (the so called "non-standard" method). Then we will investigate the effect of various model-theoretic tameness assumptions on a structure (such as stability, NIP, etc.) on the regularity of graphs definable in it (this includes algebraic and semialgebraic graphs, for example). Time permitting, we will consider Hrushovski's pseudo-finite dimension (capturing "polynomial growth", as opposed to measure theory capturing "linear growth") and some of its combinatorial applications (e.g. Erdős-Hajnal for stable hypergraphs or Shelah's pseudofinite two-cardinal theorem).

Syllabus. Ultralimits and ultraproducts, finitely additive probability measures, Szemerédi regularity lemma for graphs, regularity for hypergraphs and graph removal (after Elek-Szegedy), elements of Shelah's classification in model theory, stronger regularity lemmas for "tame" definable graphs (NIP, stable, distal, strongly minimal, etc.), Hrushovski's pseudofinite dimension and its applications (time permitting).

Prerequisites. First-order logic, basic model theory, basic measure/probability theory. All the background will be explained if necessary, please contact me if in doubt about your prerequisites.

Course text. I will follow my own notes. Some relevant references and suggested reading:
- Pillay's notes on pseudofinite model theory:
  `http://www3.nd.edu/~apillay/notes_greg-final.pdf`.
- Henry Towsner. "A Model Theoretic Proof of Szemerédi's Theorem", arXiv:1002.4456.
- My lecture notes on stability theory and on combinatorics (available at `http://www.math.ucla.edu/~chernikov`).
- Elek Gabor and Balázs Szegedy. "Limits of hypergraphs, removal and regularity lemmas. A non-standard approach", arXiv:0705.2179.
- Artem Chernikov and Sergei Starchenko. "Definable regularity lemmas for NIP hypergraphs", arXiv:1607.07701.
- Artem Chernikov and Sergei Starchenko. "Regularity lemma for distal structures", arXiv:1507.01482.
- Terence Tao. "Expanding polynomials over finite fields of large characteristic, and a regularity lemma for definable sets", arXiv:1211.2894.
- `https://terrytao.wordpress.com/` (search for the keywords on the syllabus).